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ter Haar, B; Malfiet, R

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RELATIVISTIC ASPECTS OF THE NUCLEAR MEAN FIELD IN HIGH-ENERGY NUCLEUS-NUCLEUS COLLISIONS

B. TER HAAR and R. MALFLIET

Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, The Netherlands

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The nuclear mean field as obtained within the Dirac-Brueckner approach is studied concerning its high-energy relativistic aspects. It is demonstrated that due to the different Lorentz character of the scalar and vector self-energies, which are the building blocks of the mean field, additional repulsion arises not present in non-relativistic treatments.

In order to describe nucleus-nucleus collisions in general one needs a quantum kinetic equation appropriate for non-equilibrium phenomena in nuclear matter and finite nuclei. In the equilibrium limit this equation should give the correct nuclear-matter (and finite-nuclei) properties. In the very dilute limit (low density) it should contain a free nucleon-nucleon collision term. While very general formulations [1] have been studied in the past the most practical equation which has been used in actual calculations is the Uehling-Uhlenbeck equation [2]. This equation presents the very simplest quantum extension of the classical Boltzmann equation through the incorporation of the Pauli principle in the collision term. Moreover, in studies of heavy-ion collisions it is usually complemented in an ad-hoc manner with a one-body mean field derived from the Skyrme effective interaction. The resulting equation is then called Vlasov-Uehling-Uhlenbeck (VUU) or a similar abbreviation (BUU or Landau-Vlasov) and is in principle very simple [3]. Its drawbacks however are many. First of all it includes only a small part of the many quantum effects involved i.e. through simple phase-space considerations. Secondly no microscopic derivation has been presented of the equation including this phenomenological mean field. This may lead to a double counting of the interaction since the collision term contains the full free cross section with only Pauli-blocking. Furthermore the mean field has only a dependence on

the local density and does not involve a momentum dependence which we know to be important in nucleon-nucleus scattering (optical model).

Recently we have been able to derive a quantum kinetic equation which is of the Boltzmann-Uehling-Uhlenbeck type containing both a general mean field and collision part but now inter-linked in a unique and consistent way [4]. Both these parts are expressed in terms of Brueckner G -matrix elements and the equilibrium limit of this kinetic equation (which we call the Brueckner-Boltzmann equation) corresponds exactly to the nuclear matter results obtained with the Brueckner formalism of which we presented a relativistic version (the Dirac-Brueckner approach) in ref. [5]. There it is shown that a good description of nuclear saturation and the optical model can be obtained using this approach. The collision term contains effective nucleon-nucleon "cross sections" (elastic as well as inelastic) different from the free nucleon-nucleon cross sections to which they reduce however in the dilute limit. The effective medium-corrected cross sections for nucleon-nucleon scattering as well as pion production and absorption are discussed in ref. [6].

In the self-consistent relativistic Dirac-Brueckner approach [5] a nucleon in the nuclear medium may be viewed as a bare nucleon that is "dressed" in consequence of its effective two-body interaction with the other nucleons in the medium. So far as the one-

and two-particle properties of the nuclear matter go, these can be described by a coupled set of three covariant non-linear integral equations. The self-energy $\Sigma(k)$ of the physical nucleon appears in the Dyson equation which relates the bare and physical ("dressed") nucleon propagators:

$$G(k) = G^0(k) + G^0(k)\Sigma(k)G(k). \quad (1)$$

In the Brueckner formalism this self-energy is given by a summation over all two-body interactions, which leads to the equation

$$\Sigma(k) = -i \int [\text{tr}(GF) - GF], \quad (2)$$

where the effective interaction is presented by Γ . The first term is usually referred to as the direct (or Hartree) contribution to the self-energy and the second as the exchange (or Fock) contribution. The non-relativistic first-order Brueckner theory is often referred to as the Brueckner-Hartree-Fock approach. In general the effective t -matrix Γ is a solution of the medium-dependent Bethe-Salpeter equation which can be written as

$$\Gamma = K + i \int KGG\Gamma. \quad (3)$$

The interaction K is the nucleon-nucleon interaction based on one-boson exchanges (OBE). Our OBE interaction contains π -, ω -, ρ -, ϵ -, η - and δ -exchange, and the parameters of the interaction are given in ref. [5]. A monopole formfactor $\Lambda^2/(\Lambda^2 + q^2)$ is added to the vertices. We note that the full four-dimensional Bethe-Salpeter equation is tedious to solve, and is therefore usually reduced to a covariant three-dimensional form, the so-called quasi-potential equation. The formal solution of eq. (1) in the nuclear matter rest frame is

$$G(k) = [\not{k} - m - \Sigma(k)]^{-1}. \quad (4)$$

For infinite nuclear matter, the Lorentz structure of the self-energy assumes the general form

$$\Sigma(k) = \Sigma_s(k) - \gamma^0 \Sigma_0(k) + \gamma \cdot k \Sigma_v(k), \quad (5)$$

where the tensor term has dropped out because of different symmetry relations. This structure suggests the following definitions:

$$\begin{aligned} m^*(k) &= m + \Sigma_s(k), \quad k_0^* = k^0 + \Sigma_0(k), \\ k^* &= k[1 + \Sigma_v(k)]. \end{aligned} \quad (6)$$

This enables us to express eq. (4) as $G(k) = (\not{k}^* - m^*)^{-1}$ with $\not{k}^* = \gamma^\mu k_\mu^*$. The on-shell behaviour of the effective nucleon is then given by a Dirac equation which yields a positive-energy solution

$$u(k^*, \alpha) = \left(\frac{E^* + m^*}{2m^*} \right)^{1/2} \begin{bmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{k}^*}{E^* + m^*} \end{bmatrix} \chi_\alpha, \quad (7)$$

with the energy on-shell

$$E^* = (\mathbf{k}^{*2} + m^{*2})^{1/2} = (k_0^*)^*, \quad (8)$$

from which we deduce the single-particle energy ($k_0 = E(k)$)

$$\begin{aligned} E(k) &= E^* - \Sigma_0(k) \\ &= \{ \mathbf{k}^2 [1 + \Sigma_v(k)]^2 + [m + \Sigma_s(k)]^2 \}^{1/2} \\ &\quad - \Sigma_0(k). \end{aligned} \quad (9)$$

In general m^* is momentum dependent. However it appears [5,6] that the momentum dependence of Σ is weak and Σ_v is much smaller in magnitude than either Σ_s and Σ_0 .

The nuclear mean field can be obtained from the Dirac-Brueckner expression for the self-energy $\Sigma(k)$ (eq. (5)) or the relativistic single-particle energy $E(k)$ defined in eq. (9) which in its most simple non-relativistic form can be approximated by

$$\begin{aligned} E_{\text{NR}}(k, \rho) &= m + \frac{\mathbf{k}^2}{2m} \left(1 - \frac{\Sigma_s}{m} - \frac{1}{2} \frac{\Sigma_s^2}{m^2} \right) \\ &\quad + (-\Sigma_0 + \Sigma_s) = m + \frac{1}{2} \mathbf{k}^2 / m_{\text{NR}}^* + V, \end{aligned} \quad (10)$$

illustrating the non-relativistic effective mass parameter m_{NR}^* and mean potential V . The non-relativistic effective mass m_{NR}^* is different from the relativistic one (see also ref. [5]) and its additional term arises from the second order in the expansion of (9) which also contains a \mathbf{k}^2 -dependence. The simple non-relativistic single-particle energy E_{NR} reproduces reasonably well the full relativistic one at low energies (below 300 MeV) as can be checked explicitly using our results from ref. [5]. The relativistic "definition" of a mean potential energy $U(k, \rho)$ can be given as

$$U(k, \rho) \equiv E(k, \rho) - (m^2 + \mathbf{k}^2)^{1/2}, \quad (11)$$

where we subtracted the kinetic energy of a free par-

ticle from the single-particle energy $E(k, \rho)$.

The expressions (9) and (11) give the energy of a nucleon which moves with momentum k in a nuclear medium (density ρ) at rest. We call this situation the adiabatic regime. In contrast the situation where two nuclei collide can be considered using the sudden approximation for which Σ in eq. (9) can be written as

$$\tilde{\Sigma}(k, \rho) = \Sigma'_{\text{proj}}(k, \rho, \beta) + \Sigma'_{\text{targ}}(k, \rho, -\beta), \quad (12)$$

where $\pm\beta$ is the velocity of the impinging nuclei along the z -axis in the nucleus–nucleus CM system (we consider equal nuclei). k is the momentum of a test nucleon in the nucleus–nucleus CM system. The prime in Σ' indicates the fact that this self-energy is not given in the nuclear-matter rest frame. Thus $\tilde{\Sigma}$ is the self-energy which is “felt” by a nucleon with momentum k in the CM system of the two colliding nuclei. The approximation (12) treats the influence of both nuclei on the test nucleon independent of each other and is valid for not too low values of β . In the adiabatic regime the two nuclei would have amalgamated into one dense nucleus at rest. In the sudden limit we have specific effects due to the moving pieces of nuclear matter in conjunction with their Lorentz transformation properties. The self-energies Σ'_{proj} and Σ'_{targ} in the nucleus–nucleus CM system can be calculated from the self-energy Σ in the projectile, respectively target rest system using the Lorentz transformations

$$\begin{aligned} \Sigma'_s(k) &= \Sigma_s(k'_\pm), \quad \gamma = (1 - \beta^2)^{-1/2}, \\ \Sigma'_0(k) &= \gamma[\Sigma_0(k'_\pm) \pm \beta(\Sigma_v(k'_\pm))_L], \\ (\Sigma'_v(k))_L &= \gamma[(\Sigma_v(k'_\pm))_L \pm \beta\Sigma_0(k'_\pm)], \\ (\Sigma'_v(k))_T &= (\Sigma_v(k'_\pm))_T, \end{aligned} \quad (13)$$

where we used the notation $\Sigma_v \equiv \mathbf{k}\Sigma_v$ (because of rotational invariance in uniform nuclear matter) and the indices L, T denote respectively the parallel and perpendicular components of Σ_v with respect to the z -axis. The momenta $(k'_\pm)_\mu$ in the projectile, respectively target system are obtained from the momentum $k_\mu = (\mathbf{k}, k_0)$ in the nucleus–nucleus CM system through a similar Lorentz transformation:

$$\begin{aligned} (\mathbf{k}'_\pm)_L &= \gamma(\mathbf{k}_L \pm \beta k_0), \quad k_0 = E(k, \rho), \\ (\mathbf{k}'_\pm)_T &= \mathbf{k}_T, \quad (k'_\pm)_0 = \gamma(k_0 \pm \beta \mathbf{k}_L). \end{aligned} \quad (14)$$

Remark that the Lorentz transformations (14) themselves depend on the single-particle energy $E(k, \rho)$ calculated in the sudden limit:

$$E(k, \rho) = [(m + \tilde{\Sigma}_s)^2 + \mathbf{k}^2(1 + \tilde{\Sigma}_v)^2]^{1/2} - \tilde{\Sigma}_0, \quad (15)$$

with $\tilde{\Sigma}_s$, $\tilde{\Sigma}_v$ and $\tilde{\Sigma}_0$ defined in (12) and (13).

In the adiabatic limit (test nucleon with momentum k moves in stationary nuclear medium of density ρ) both increasing density ρ and increasing momentum k of the test nucleon will give rise to a repulsive mean field. However, in the sudden limit (test nucleon with momentum k finds itself in two moving pieces of nuclear matter) an additional effect comes into play because of the specific transformation properties of the fields Σ_s and Σ_0 (see eq. (13)). Roughly speaking, in the adiabatic limit the nuclear mean-potential field goes as $\Sigma_s - \Sigma_0$ while in the sudden limit it goes as $\Sigma_s - \gamma\Sigma_0$. Since $\Sigma_s - \Sigma_0$ is relatively small but Σ_s and Σ_0 are large (and have equal negative sign) the effect of $\gamma\Sigma_0$ ($\gamma > 1$, $\Sigma_0 < 0$) produces additional repulsion. One can say that in the sudden limit with increasing β the scalar and vector fields decouple from each other. This is a genuine effect of relativity and it might produce a quite interesting (and very repulsive) behaviour with increasing bombarding energy. This particular behaviour is absent in non-relativistic considerations where one does not differentiate between Lorentz scalars and vectors. Indeed using the simple non-relativistic form (10), with m_{NR}^* and V given (either calculated non-relativistically or fitted to optical potential data), fails to reveal this behaviour. In the following we illustrate both the adiabatic and sudden limit.

From the Σ_s , Σ_0 results the single-particle energy can be calculated (nucleon with momentum k in nuclear matter at rest) and successively the resulting mean field $U(k, \rho)$, eq. (11) can be constructed which contains all medium effects (including also effects of the effective nucleon mass). The values for Σ_s , Σ_0 and Σ_v as a function of k and nuclear matter density ρ can be found in refs. [5,6]. In ref. [6] they are given up to energies of 1 GeV. In ref. [5] we have also shown that the resulting $U(k, \rho)$ at $\rho = \rho_0$ are similar to the ones obtained through a non-relativistic Brueckner calculation [7]. However for $\rho \geq \rho_0$ they differ substantially since the non-relativistic one does not reproduce the correct saturation behaviour while in the Dirac–Brueckner approach saturation is cor-

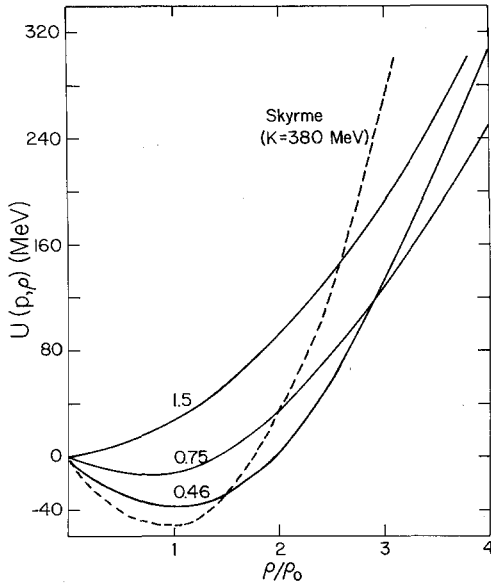


Fig. 1. The mean potential energy $U(p, \rho)$ in the adiabatic limit calculated in the Dirac-Brueckner approach for three different densities ρ and single-particle momenta p (0.46, 0.75, 1.5 GeV/c). The dashed line is the "stiff" Skyrme mean potential energy (eq. (16)).

rectly reproduced. In fig. 1 we display $U(k, \rho)$ in the adiabatic limit for three different values of the nucleon momentum k (0.46, 0.75, 1.5 GeV/c) as a function of ρ/ρ_0 where ρ_0 is the nuclear saturation density. The dotted curve represents a static "Skyrme" parametrization:

$$U_{\text{SK}}(\rho) = A\rho + B\rho^2, \quad (16)$$

which corresponds to a single-particle energy density $\epsilon(\rho) = \epsilon_F(\rho) + U_{\text{SK}}(\rho)$ where $\epsilon_F(\rho)$ is the Fermi energy and through fitting A and B gives the correct saturation properties ($\epsilon(\rho) = -16$ MeV at $\rho = \rho_0$) and a compression modulus of $K = 380$ MeV. This particular potential has been used extensively by the Frankfurt group [8] in VUV calculations. It is clear from our comparison in fig. 1 that the momentum dependence which is absent in (16) (since it is averaged over a Fermi momentum distribution) is very important. In fact at $k = 1.5$ GeV/c and $\rho = \rho_0$ we have as much repulsion as the "Skyrme" static mean field at $\rho = 2\rho_0$. This observation was made already some time ago [9] and its implications have been substantiated very recently in calculations using a modified VUU [10]. Consequently a soft equation of state including a momentum-dependent mean field

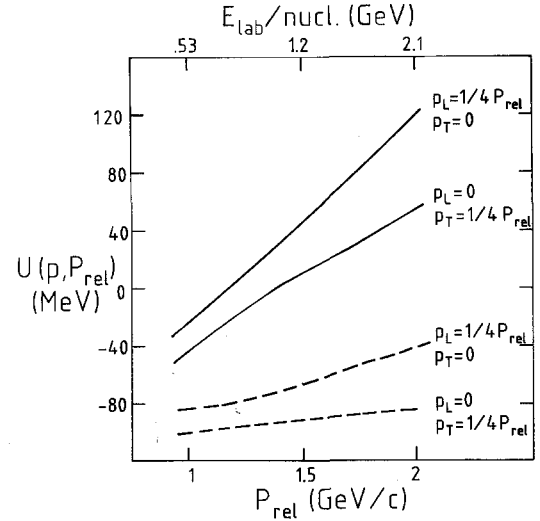


Fig. 2. The mean potential energy $U(p, \rho)$ calculated in the Dirac-Brueckner approach for two moving pieces of uniform nuclear matter at normal density ($\rho = \rho_0$). Their relative CM momentum is given by P_{rel} (GeV/c). The single particle momentum of the test nucleon in the CM system of the two moving pieces is variably fixed as displayed in the figure (p_L , longitudinal component; p_T transversal component). The upper scale gives an indication of the corresponding energy per nucleon of the moving nuclear matter piece in the laboratory system (where the other piece is at rest).

can be mocked up by a hard equation of state without momentum dependence. The correct description however, always contains a momentum-dependent mean field and therefore the evidence for a hard equation of state of the form (16) seems to be fortuitous. We remark also that the density and momentum dependence of $U(p, \rho)$ exhibits a non-monotonic behaviour. This is due to the particular momentum dependence of the underlying fields Σ_s and Σ_0 . This feature is absent in relativistic mean field results [11], where no such momentum dependence exists, and which show also a more regular behaviour.

Moreover, colliding nuclei will initially always go through the sudden-limit situation and this might even increase the repulsiveness (and effectively soften the equation of state further). In fig. 2 we show some of the consequences following from the decoupling of Σ_s and Σ_0 in the sudden limit. Two nuclei collide with each other and in their CM system they have a relative momentum P_{rel} along the z -axis. We take a test nucleon which has momentum

$|\mathbf{k}| = |\mathbf{P}_{\text{rel}}|/4$ in the nucleus-nucleus CM system. This momentum represents an average value of the possible values which can be attained initially by a participant nucleon in the collision of two nuclei. We choose this momentum either along the z -axis or perpendicular to it. Then we calculate iteratively the single particle energy $E(k, \rho)$ in eq. (15) with $\tilde{\Sigma}$ obtained from (12)–(14). The resulting $U(k, \rho)$ (see eq. (11)) are displayed as full lines. The dotted lines have been calculated similarly except that now we have omitted the explicit Lorentz transformations (13) on the Σ_s and Σ_0 but keeping the transformations in (14) for the momenta. The difference between full and dotted lines is thus an indication of the importance of the Lorentz-character of the fields Σ_0 and Σ_s , a feature absent in non-relativistic calculations. Indeed, using expression (10) with given V and m_{NR}^* , the change from one frame to another one is only reflected in the transformation properties of \mathbf{k}^2 . Obviously from fig. 2 we conclude that there will be an additional repulsive effect besides the ones linked directly to momentum and density dependence. This might be an important ingredient in considerations of stopping power at high bombarding energies.

The difference between the mean fields corresponding to longitudinal and transversal momentum of the test nucleon is also interesting. At low bombarding energies this will produce a net transversal flow since the repulsion is greater in longitudinal direction and test nucleons are produced in an isotropic fashion. At high energies the situation is less clear since now due to forward-backward-peaked cross sections more nucleons are produced in longitudinal direction.

In conclusion we have shown that a relativistic description of the nuclear mean field as can be obtained in the Dirac-Brueckner approach contains certain aspects completely absent in its non-relativistic counterpart. Similar observations were made by other authors [12]. These effects will be especially important at high bombarding energies and be of interest for instance concerning the deduction of compression energy from pion yields [8,14]. However a consistent treatment should be based also on a relativistic kinetic equation which not only calculates the scalar nucleon density matrix but all its Lorentz-Dirac invariants (scalar, vector, tensor, ...)

in a similar fashion as the decomposition (5) for the self-energy [13]. For nucleons (neglecting anti-nucleons) this leads to 16 coupled equations corresponding to the 16 invariants $\{1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_5 \gamma_\mu, \gamma_5 \gamma_\mu \not{p}^\mu\}$. The semi-classical limit of these equations for the scalar density matrix then corresponds to a kind of Boltzmann equation, which can be generalised such as to contain the Dirac-Brueckner effective interaction as its dynamic input [15]. Clearly the formulation of such a theory is very important and we are presently pursuing this goal.

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